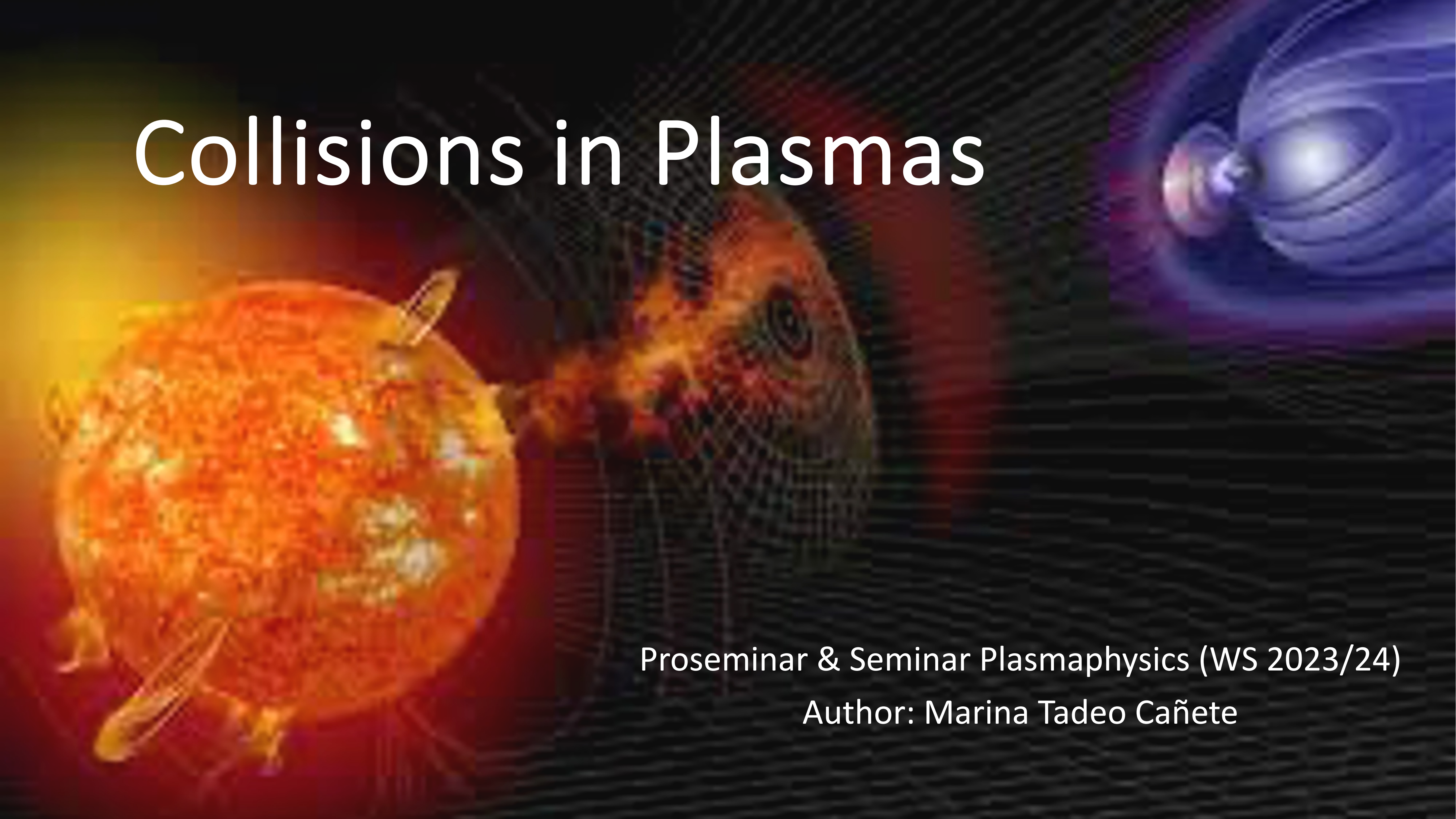

Collisions in Plasmas

Proseminar & Seminar Plasmaphysics (WS 2023/24)
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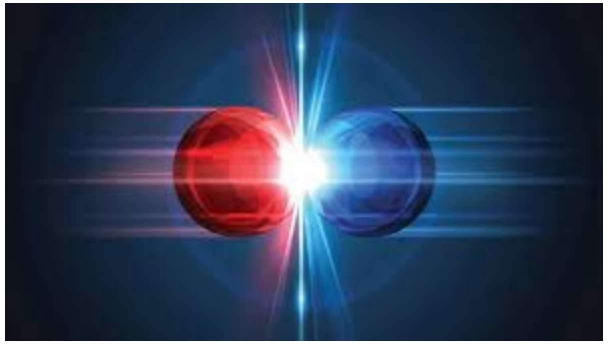
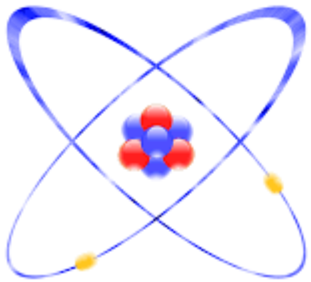
- A **plasma** is a state of matter in which the gas is fully or partially ionized and coexist in a mixture of constantly moving charged particles.
- Collisions between these particles are fundamental to understanding and controlling plasmas in various applications.
- The types of particles present in the plasma (ions, electrons and also neutrals) interact with each other via collisions.





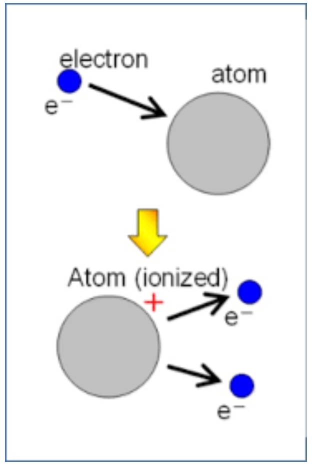


Figure. Collisions in charged particles

Collisions in plasmas vs to neutral gases

- In **neutral gases**:

  Collisions involve neutral atoms or molecules interacting through short-range forces (Van der Waals/London dispersion forces).

- In **plasmas**:

  Charged particles in plasmas interact through long-range Coulomb forces.

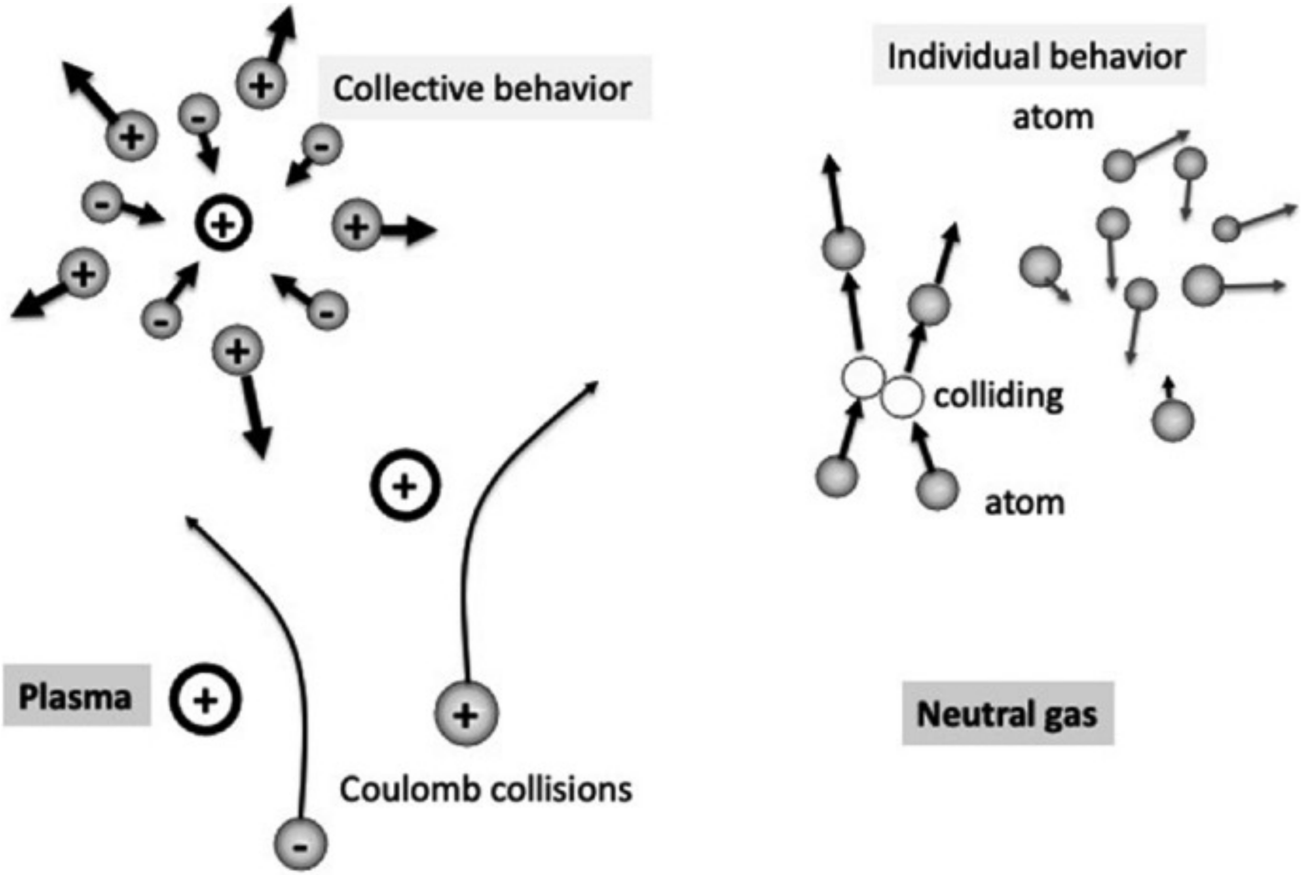


Figure. Introduction to Plasma

Collisions in plasmas vs to neutral gases

	PLASMAS	NEUTRAL GASES
Nature of Particles	Free electrons and ions.	Short-range elastic collisions
Effects of Collisions	Changes in the kinetic energy and velocity of charged particles. (Coulomb interactions)	Changes in the direction and speed of particles, but typically do not significantly affect their temperature or energy
Time and Length Scales	Plasmas can vary depending on plasma density, temperature, and other properties	These scales are generally shorter. Collision frequency (dens*v*σ) and the mean free path (1/σ*n)

Rutherford cross section

- The scattering of charged particles by matter is called Coulomb or Rutherford scattering when it takes place at low energies, where only the Coulomb force is important.

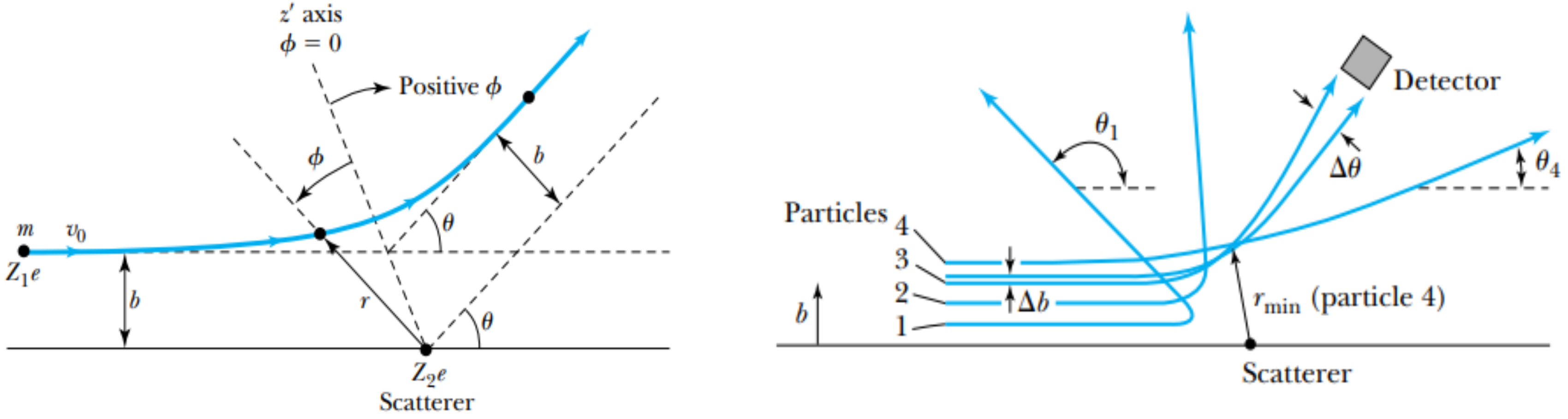


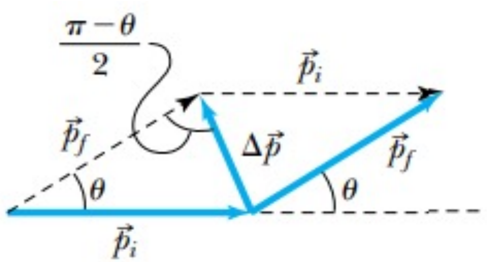
Figure .Representation of Coulomb or Rutherford scattering.

Parameters:

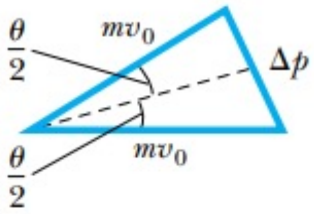
- A charged particle of mass **m**, charge **Z_1e**, and speed **v_0** is incident on the scatterer of charge **Z_2e**
- The distance **b** is called the classical impact parameter (it is the closest distance of approach)
- The scattering angle θ between the incident beam direction and the direction of the deflected particle

Fundamental relationship between the impact parameter b and scattering angle θ for the Coulomb force

- We define that the change in momentum is equal to the impulse $\Delta\vec{p} = \int \vec{F}_{\Delta p}\, dt$



- Because $p_f \approx p_i = mv_0$ the magnitude Δp is now $\Delta p = 2mv_0 \sin\frac{\theta}{2}$
- Now if we define the Coulomb force F along the instantaneous direction of the position vector r: $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Z_1 Z_2 e^2}{r^2}\hat{e}_r$ → $F_{\Delta p} = F\cos\phi$



- Substituting the magnitudes we obtain: $\Delta p = 2mv_0 \sin\frac{\theta}{2} = \int F\cos\phi\, dt = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\int \frac{\cos\phi}{r^2}\, dt$
- Also, the instantaneous angular momentum must be conserved, so: $mr^2\frac{d\phi}{dt} = mv_0 b$
- If we substitutes r^2: $2mv_0 \sin\frac{\theta}{2} = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\int \frac{\cos\phi}{v_0 b}\frac{d\phi}{dt}\, dt$ → $\frac{8\pi\epsilon_0 m v_0^2 b}{Z_1 Z_2 e^2}\sin\frac{\theta}{2} = \int_{-(\pi-\theta)/2}^{+(\pi-\theta)/2} \cos\phi\, d\phi = 2\cos\frac{\theta}{2}$
- Finally, solving the integration we obtain the relationship between b and θ **:** $b = \frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K}\cot\frac{\theta}{2}$

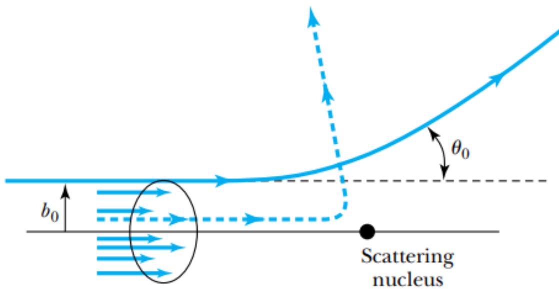
Rutherford cross section. Step 2

- For the case of Coulomb scattering, we denote the cross section by the symbol $\sigma = \pi b^2$
- **The cross section σ** is related to the probability for a particle being scattered by a nucleus. $nt = \frac{\rho N_A N_M t}{M_g} \frac{\text{atoms}}{\text{cm}^2}$
- The probability of the particle being scattered is equal to the total target area exposed for all the nuclei divided by the total target area A. The fraction of incident particles scattered by an angle of θ or greater is:

$$f = \frac{\text{target area exposed by scatterers}}{\text{total target area}} = \frac{ntA\sigma}{A}$$

$$= nt\sigma = nt\pi b^2$$

$$f = \pi nt \left(\frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K} \right)^2 \cot^2 \frac{\theta}{2}$$



- The fraction of the incident particles scattered between θ and d θ is df

$$df = -\pi nt \left(\frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K} \right)^2 \cot \frac{\theta}{2} \csc^2 \frac{\theta}{2} \, d\theta$$

The important points:

$$df = -\pi n t \left(\frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K} \right)^2 \cot\frac{\theta}{2} \csc^2\frac{\theta}{2}\, d\theta$$

1. Proportional to the square of the atomic number of both the incident particle (Z_1) and the target scatterer (Z_2).
2. The number of scattered particles is inversely proportional to the square of the kinetic energy K of the incident particle
3. The scattering is inversely proportional to the fourth power of sin(θ/2), where θ is the scattering angle.

General definitions

- For a moving particle with atomic density n **the average number of collision processes** in a pathlength interval Δs is given by

$$\Delta N_c = n\sigma\Delta s$$

- One collision in average defines the collisional **mean free path length**

$$\lambda_c = \frac{1}{n\sigma}$$

- From this, the **mean collision time** for a particle with velocity v is

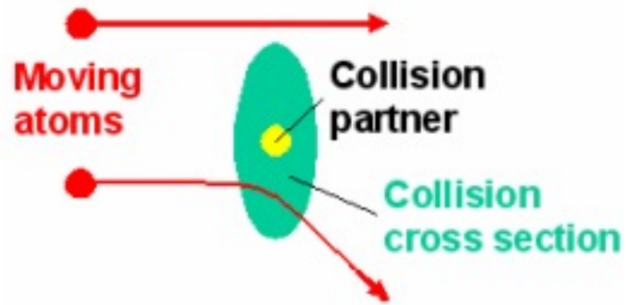
$$t_c = \frac{\lambda_c}{\upsilon} = \frac{1}{n\upsilon\sigma}$$

- If the moving particle belongs to an ensemble with a distribution of velocities, it can be defined the collisional **"rate coefficient“**

$$\langle\sigma\upsilon\rangle = \frac{\int \upsilon^3\sigma(\upsilon)f(\upsilon)d\upsilon}{\int \upsilon^2 f(\upsilon)d\upsilon}$$

- Then it can be obtained the **collision frequency**

$$\nu_c = n\langle\sigma\upsilon\rangle$$



Plasma Frequency

- It is the most fundamental timescale in plasma physics.

$$\Pi = \left(\frac{n\,e^2}{\epsilon_0\,m}\right)^{1/2}$$

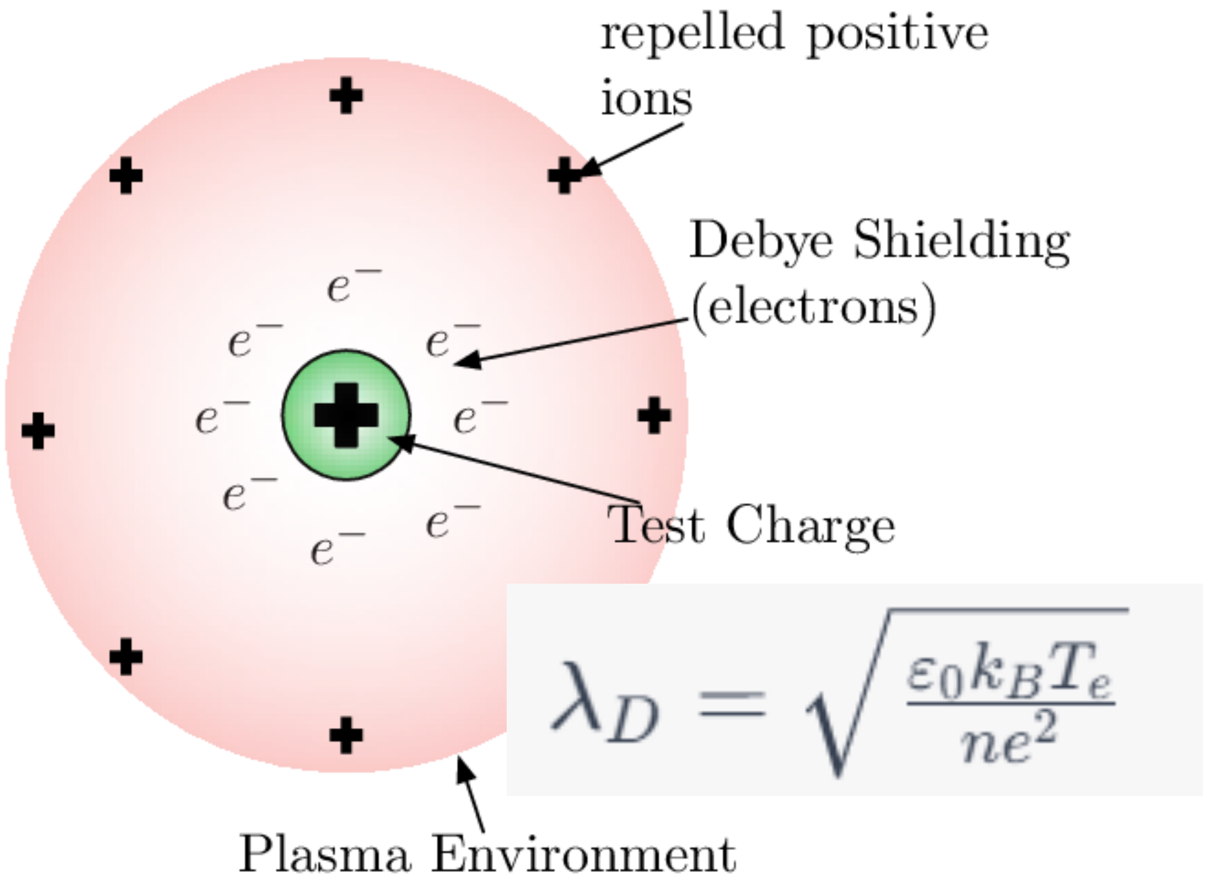
- Plasma oscillations are observed only when the plasma system is studied over time periods longer than the plasma period τp=2*π/л .
- Observations over length scales L shorter than the distance travelled by a plasma particle during a plasma period will also not detect plasma behavior. This distance is the called Debye length.

- Our idealized system can usefully be considered to be a plasma only if:

$$\frac{\lambda_D}{L} \ll 1, \qquad \frac{\tau_p}{\tau} \ll 1.$$

Debye length and Debye Cut off

- Characterizes the spatial extent over which electric fields are effectively shielded or screened by charged particles within a plasma.
- It is a measure of the range of influence of these charges on the electric potential in the plasma.
- The Debye Cut Off marks the point beyond which long-range interactions in a plasma are attenuated.
- Beyond this distance, the influence of individual charges becomes significantly reduced due to the collective behavior of the plasma.



$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{n e^2}}$$

Coulomb logarithm

- Quantifies the strength of Coulomb interactions in a plasma. It represents the ratio of the maximum impact parameter to the average impact parameter in Coulomb collisions and influences the scattering cross section.
- It determines the probability of a collision causing a deviation in the trajectories of charged particles.
- Usually **bmax** is given by the Debye length
  - On length scales ≳λD, electric fields from individual particles get cancelled out due to screening effects. Consequently, Coulomb collisions with impact parameters ≳λD will rarely occur.
- The inner impact parameter **bmin** requires more nuance. (90° deflection angle)

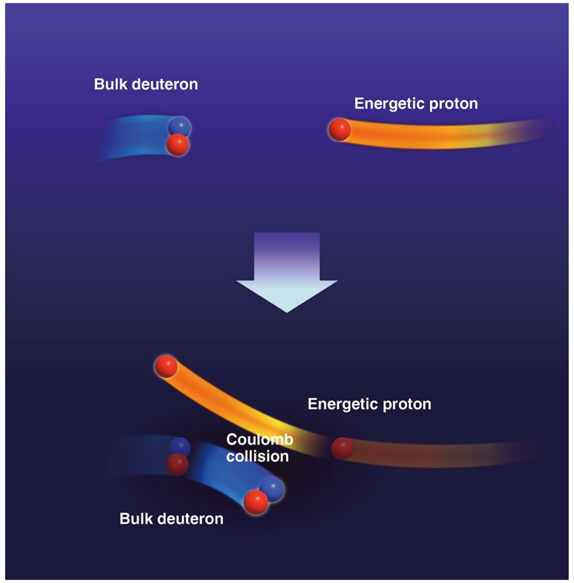


Figure. Diagram of Coulomb Collisions

$$\ln \Lambda \equiv \ln \left(\frac{b_{\max}}{b_{\min}} \right).$$

Coulomb logarithm

- Coulomb Logarithm derived from calculating the ratio between small angle and large angle collisions.
- To do this we need the impact parameter that leads to 90 degree collisions:

$$b_{90} = \frac{q_1 q_2}{4\pi\epsilon_0}\frac{1}{\mu_r u^2} \approx \frac{q_1 q_2}{4\pi\epsilon_0}\frac{1}{3T} = \frac{Z_1 Z_2}{12\pi\lambda_D^2 n}.$$

- Then, the ratio of small to large angular scatter is therefore:

$$\Lambda^2 = \frac{\lambda_D^2 - b_{90}^2}{b_{90}^2} = \frac{\lambda_D^2}{b_{90}^2} - 1 \approx \frac{\lambda_D^2}{b_{90}^2} = \left(\frac{12\pi}{Z_1 Z_2}\right)^2 \lambda_D^6 n^2 = \left(\cot\frac{\chi_{min}}{2}\right)^2$$

- It plays an important role in all production processes in plasma physics and can also be written as:

$$\ln\Lambda = \ln\left(\cot\frac{\chi_{min}}{2}\right) = \ln\left(\cos\frac{\chi_{min}}{2}\right) - \ln\left(\sin\frac{\chi_{min}}{2}\right) \approx -\ln\left(\sin\frac{\chi_{min}}{2}\right)$$

- Collisions between two charged particles where the interaction is governed solely by the electric fields from the two particles.
- Usually result in small deflections of particle trajectories.
    - High impact parameter collisions occur much more frequently than low impact parameter collisions
- The min and max impact parameters represent the range of distances of closest approach.
- While a typical Coulomb collision results in only a slight change in trajectory, the effects of these collisions are cumulative, and it is necessary to integrate over the range of impact parameters to account for the effects of Coulomb collisions throughout the plasma. So the Coulomb logarithm accounts for the range in impact parameters for the different collisions.

- Defining the average distance between particles $r_d \equiv n^{-1/3}$ and a distance of closest approach $r_c \equiv \frac{e^2}{4\pi\,\epsilon_0\,T}$
- By balancing the one-dimensional thermal energy of a particle against the repulsive electrostatic potential of a binary pair and defining the plasma parameter, we obtain:

$$\frac{1}{2}\,m\,v_t^2 = \frac{e^2}{4\pi\,\epsilon_0\,r_c}$$

$$\Lambda = \frac{4\pi}{3}\,n\,\lambda_D^3$$

$$\Lambda = \frac{\lambda_D}{3\,r_c} = \frac{1}{3\sqrt{4\pi}}\left(\frac{r_d}{r_c}\right)^{3/2} = \frac{4\pi\,\epsilon_0^{3/2}}{3\,e^3}\,\frac{T^{3/2}}{n^{1/2}}.$$

- This dimensionless parameter is obviously equal to the typical number of particles contained in a Debye sphere.

-**The case Λ<<1**: in which the Debye sphere is sparsely populated, corresponds to a strongly coupled plasma (tend to be cold and dense).

-**The case Λ>>1 :** in which the Debye sphere is densely populated, corresponds to a weakly coupled plasma (tend to be diffuse and hot).

Plasma	$n(\mathrm{m}^{-3})$	$T(\mathrm{eV})$	$\Pi(\mathrm{sec}^{-1})$	$\lambda_D(\mathrm{m})$	Λ
Solar wind (1AU)	10^7	10	2×10^5	7×10^0	5×10^{10}
Tokamak	10^{20}	10^4	6×10^{11}	7×10^{-5}	4×10^8
Interstellar medium	10^6	10^{-2}	6×10^4	7×10^{-1}	4×10^6
Ionosphere	10^{12}	10^{-1}	6×10^7	2×10^{-3}	1×10^5
Inertial confinement	10^{28}	10^4	6×10^{15}	7×10^{-9}	5×10^4
Solar chromosphere	10^{18}	2	6×10^{10}	5×10^{-6}	2×10^3
Arc discharge	10^{20}	1	6×10^{11}	7×10^{-7}	5×10^2

Table 1.1: Key parameters for some typical weakly coupled plasmas.

Conclusions of collisions in plasma

- Collisions between charged particles in a plasma differ fundamentally from those between molecules in a neutral gas because of the long range of the Coulomb force.
- The collision times (inversely proportional to the plasma density) and free mean path (depends on the cross section and particle density) in plasmas are critical parameters that define the timescales and lengthscales of collisional processes.
- The collision frequency, measures the frequency with which a particle trajectory undergoes a major angular change due to Coulomb interactions with other particles. Coulomb collisions are, in fact, predominately small angle scattering events. In a plasma, the Debye length introduces a characteristic scale that influences collisional dynamics.
- While collisions are crucial to the confinement and dynamics of neutral gases, they can under some conditions play a less important role for some physical properties, as hot plasmas can often be considered to be collisionless.

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